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ABSTRACT

Developmental patterns in one mode of formal reasoning (probabilistic reasoning) were examined in this research study, specifically: (1) to determine if developmental patterns of probabilistic reasoning could be confirmed using a large, diverse sample of subjects, and if they could be similar despite variations in the complexity of the reasoning required to solve a problem; (2) to determine if variations in problem context led to variations in response patterns; and (3) to determine if developmental patterns of probabilistic reasoning were similar for subjects of similar reasoning ability regardless of education level. Data from 2282 subjects in grades 6-13 were collected using Tobin and Capie's Test of Logical Thinking (TOLT), form A (N=1627) and form B (N=655). For each of the four items, the proportion of students obtaining correct responses was linearly related to their formal reasoning ability and a measure of the TOLT. Two common approaches were identified for individuals attempting to use probabilistic reasoning to solve problems. (DS)

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PATTERNS OF REASONING : PROBABILISTIC REASONING

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PATTERNS OF REASONING : PROBABILISTIC REASONING

The study of formal reasoning ability has become increasingly important as educators and psychologists endeavor to understand learning in adolescence and adulthood. Recent studies have tended to focus on five reasoning modes : proportional reasoning; controlling variables; probabilistic reasoning; correlational reasoning; and combinatorial reasoning. In science education, most investigators have researched aspects of four of these: proportional reasoning; combinatorial reasoning; controlling variables; and correlational reasoning (e.g. Lawson, Adi, & Karplus, 1979; Lawson, 1979; Linn & Pulos, 1979). Few studies have focussed on the development of probabilistic reasoning and implications for science teaching.

Extensive research on the development of probabilistic reasoning is reported by Inhelder and Piaget (1975, 1958), and Fischbein (1975). However, the generalizability of the reasoning patterns identified and described has not been demonstrated: from adolescence through adulthood; with variations in context; and variations within the reasoning modes themselves. Traditionally, formal reasoning ability was assessed in an individual interview with standard problem protocols presented in a flexible question sequence based on models reported by Inhelder and Piaget. The majority of these studies have been characterized by relatively small samples, and the assessment of few reasoning modes, each with one interview task. Thus, this study was planned to examine developmental patterns in probabilistic reasoning for a large and diverse sample of subjects.

Four concerns provided a focus for the study:

1. to determine if developmental patterns of probabilistic reasoning could be confirmed using a large, diverse sample of subjects;

2. to determine if developmental patterns of probabilistic reasoning were similar despite variations in the complexity of the reasoning required to solve a problem;

3. to determine if variations in problem context led to variations in response patterns from subjects; and

4. to determine if developmental patterns of probabilistic reasoning were similar for subjects of similar formal reasoning ability regardless of educational level.

Procedures

Data from 2399 subjects in college classes and grades 6-12 were collected using the Test of Logical Thinking (TOLT) (Tobin & Capie, 1981). Form A of the TOLT was used to assess 1627 subjects and Form B was used to assess 772 subjects. The psychometric characteristics, item format, and the existence of parallel forms made the TOLT particularly well suited to this investigation. Capie and Tobin (1980)

reported internal consistency reliability coefficients for each form of the TOLT to be greater than .8. Evidence was also reported of high correlations between performance on form A of the TOLT and performance on five standard Piagetian interviews. Other validity data that are reported argue for construct and criterion-related validity for each measures. The correlation between performance on each form of the TOLT was approximately .8.

In each form of the TOLT, subjects are presented with ten items, two items for each of five formal reasoning modes. Two problems in each form of the TOLT involve probabilistic reasoning and were the data source for this investigation. Each of these items required subjects to calculate the odds of selecting one object on the first occasion from a number of objects. The items are presented in Figures 1-4. In each test form the first probability item has two sets of objects from which to choose. In each item the subject is required to choose a correct answer from five alternatives, and then select a justification for the choice from five that are provided. Thus, twenty-five possible combinations exist for each item. The most common combinations for each problem were selected for additional study. Infrequently occurring responses were not investigated in this study.

Insert Figure 1, Figure 2, Figure 3, Figure 4 about here

Subjects were ranked on the basis of total performance on the TOLT which provided a measure of formal reasoning ability. Response

patterns were separately analyzed at each level of formal reasoning ability defined by scores on the TOLT (0-10). Similar analyses were conducted for samples at three educational levels: college students; high school students; and middle school students.

Because the distributions of the TOLT scores varied with educational level, two levels corresponding to scores of 2,3 and 4,5 on the TOLT were chosen to compare response patterns provided by subjects at college, high school, and middle school. Only at these levels were expected cell frequencies sufficiently high to avoid violating assumptions underlying the chi-squared test. Analyses were only conducted for the two probabilistic reasoning items from form A of the TOLT.

Results

The results of the analyses are presented in two sets of tables. Table 1 - Table 4 contain response patterns for each total score on the TOLT; and Table 5 and Table 6 contain response patterns for each educational level for probabilistic reasoning items on form A of the TOLT.

Insert Table 1 - Table 6 about here

Discussion

The most common incorrect answer for the Vegetable Seeds problem

was 1/6 with the justification that one bean seed must be chosen from a total of six seeds. Another frequently occurring incorrect response was 1/3, with either of two reasons provided as support for the answer: one bean seed needs to be selected from a total of three; and one half of the seeds are bean seeds. The results reported in Table 1 show that the correct answer of 1/2 became the most common response at a score of 4 on the TOLT.

A similar response pattern was observed for the Swiss Train item from Form B of the TOLT. In this case, however, the most common incorrect response was 1/3 with justifications parallel to those provided for the Vegetable Seeds item. A response of 1/6 was also a common answer provided for the Swiss Train item. The results in Table 2 show that the correct answer of 1/2 became the most common response at a score of 4 on the TOLT.

The most common incorrect responses on the Flower Seeds item were 1/7 and 1/21. The results in Table 3 show that 1/21 was the most common response for TOLT scores of 0-3. For higher scores on the TOLT the correct answer became the most common response. As was the case with the Vegetable Seeds and Swiss Train items, two reasons were frequently volunteered in support of 1/7 : seven of the twenty one seeds will produce red flowers; and one red seed needs to be picked from a total of seven red seeds. In both cases subjects only considered the favorable response set when estimating the odds.

The correct response of 1/3 for the Coins and Rings item became

the most common response at a score of 5 on the TOLT. Incorrect responses paralleled those provided for the Flower Seeds item. The most common incorrect response for the Coins and Rings item was 1/7. Two justifications were frequently provided for this answer : one gold object needs to be selected from a total of 7 gold objects; and 7 of the 21 objects in the sack are made from gold. No clear pattern governed which alternative was favored for lower levels of formal reasoning ability, however for scores greater than 3 on the TOLT the explanation that 7 of the 21 objects in the sack were gold was volunteered most frequently. A response of 1/21 was frequently provided for subjects scoring from 0-8 on the TOLT, however, this response was not the most frequently occurring response at any score on the TOLT.

The results for the Vegetable Seeds item suggested very similar response patterns for subjects at each educational level. Five response patterns were examined at three educational levels. Chi-squared values of 13.96 (TOLT = 2) and 12.15 (TOLT = 3) were not statistically significant (8df, $p > 0.05$). This result supported a similarity of response patterns across educational levels. As the educational level was varied, the response patterns tended to be similar for each developmental level.

The results obtained for the Flower Seeds item (Table 6) indicated that educational level was associated with response patterns at each score on the TOLT (i.e. TOLT = 2,3 and TOLT = 4,5). However, a value of approximately 0.2 for Cramer's V in each case suggested a weak

relationship. Planned contrasts using the Dunn-Bonferroni procedure indicated that a greater proportion of pupils at college level selected the correct response pattern than pupils at the middle school level. A similar trend was observed for scores on the TOLT of 2,3 and 4,5. Post-hoc investigation did not reveal other pairwise differences of practical importance.

The results obtained for the Vegetable Seeds item conflict with those obtained with the Flower Seeds item. One possible explanation is that College pupils were able to solve the more difficult Flower Seeds item algorithmically. This explanation is plausible since many of the College pupils had been taught probability in mathematics courses. This was usually not the case with middle school students.

One limitation of this study was that subjects were provided with only twenty-five response options. An open response format may have produced greater diversity or other common patterns. However, with these limitations, two common approaches were identified for individuals attempting to use probabilistic reasoning to solve problems. The two incorrect approaches that tended to occur were:

1. subjects considered the entire set of objects, but did not consider the effects of multiple elements from the favorable outcome set; or

2. subjects only considered selection from the favorable outcome set and ignored the presence of other sets of objects.

The greatest implications of this research are for instruction. The study has indicated a large proportion of pupils at each educational level who are not able to quantify the odds in simple problem situations. In view of research reported by Fischbein (1975) suggesting that probabilistic reasoning may be promoted through instruction, the results of this study may have implications for teachers at all levels. When problems need to be solved requiring probabilistic reasoning, difficulties similar to those described above are likely to be encountered. Pre-instruction that highlights the shortcomings of the above types of reasoning may assist in improving reasoning ability and enhance understanding of concepts that are probabilistic in nature.

The content for each TOLT item was non-scientific. Changes in the context from Form A to Form B produced few noticeable changes in response selection. Reasoning patterns tended to be independent of the problem setting. The implications of this finding relate to assessment and use of probabilistic reasoning. In the first instance, researchers can be confident in using diverse non-technical content to assess reasoning modes since they appear to be stable over quite different content areas (e.g. Vegetable seeds compared to passengers in a train). From an instructional perspective, reasoning patterns appear to be generalizable, allowing skills developed in one context to be applied to problem solving in other contexts.

The implications for teaching science are less obvious and need to

be inferred. Learning science concepts such as the gas laws, Mendelian genetics, and atomic structure certainly require some understanding of probability, however, pupils are not usually required to calculate the odds of a particular event occurring. From a logical perspective, an ability to estimate the odds in simple problem situations is likely to be closely related with an ability to understand explanations of probabilistic phenomena. Many science activities advocated for middle school pupils utilize probabilistic concepts when group data are averaged to obtain estimates of "population" values. For example in an investigation of the relationship between physical exercise and pulse rate, variations in human attributes may be controlled by using one person from each group of three as a data source in an investigation. If physical exercise is designed and manipulated in the same way throughout the class, then class mean values can be used in hypothesis testing. An understanding of the basis for using mean values necessitates an understanding of the probabilistic nature of human behavior.

Possible differences in reasoning patterns between college pupils and middle school pupils at the same cognitive level need to be investigated further. Since a significant proportion of college pupils have difficulty in using probabilistic reasoning to solve problems, priority should be given to determining whether reasoning can be enhanced by instruction. For example, if materials centered instruction on probability can be used to enhance probabilistic reasoning, then learning experiences can be prescribed for pupils in need of assistance.

Conclusion

This study has been concerned with the quantification of probability concepts. The analyses of response patterns have provided insights into the way probabilistic reasoning is pursued. The two most common quantitative reasoning problems occur when pupils only consider the number of objects in the favorable response set, and when the number of objects to be drawn is compared to the total number of objects. These problems were evident from comparisons of response patterns from Form A and Form B of the TOLT where the reasoning pattern required to solve a problem was controlled. The problems were also apparent in comparisons of items within each form of the TOLT. In these cases the context of the problem was such that one problem was more complex than another. Generally, the results also suggested similar response patterns for subjects at each educational level. Differences between items were attributed to prior instruction in probability for College pupils.

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Table 1

Response Patterns for the Vegetable Seeds Problem

Score on the TOLT	Correct response	One selected from total set	One selected from favorable response set	b-4
0	0.0	57.1	3.3	4.3
1	6.0	36.4	13.2	4.0
2	12.1	44.7	7.1	5.0
3	10.5	28.6	9.8	6.0
4	37.5	19.7	6.6	9.9
5	50.3	18.1	8.8	3.5
6	54.3	16.5	6.8	5.1
7	69.9	9.8	1.2	6.1
8	81.0	5.1	1.9	2.5
9	91.0	3.2	1.3	1.3
10	100.0	0.0	0.0	0.0

Table 2

Response Patterns for the Swiss Train Problem

Score on the TOLT	Correct response	One selected from total set	One selected from favorable response set	b-4
1	0.0	16.7	58.3	0.1
2	0.0	7.1	7.1	0.0
3	18.6	16.3	18.3	11.6
4	32.1	17.0	15.1	13.2
5	36.6	0.0	4.9	9.8
6	43.5	10.9	12.0	13.0
7	59.5	9.5	7.1	7.1
8	71.1	3.3	9.1	5.8
9	85.3	3.4	1.7	4.3
10	100.0	0.0	0.0	0.0

Table 3

Response Patterns for the Flower Seeds Problem

Score on the TOLT	Correct response (%)	One selected from total set (%)	One selected from favorable response set (%)	
			(a)	(b)
0	0.0	40.6	12.1	6.6
1	1.0	36.9	15.4	13.4
2	7.8	27.0	9.2	18.4
3	19.5	19.5	9.8	9.0
4	36.2	7.9	11.2	6.6
5	50.3	6.4	5.8	8.8
6	54.0	7.3	9.1	9.7
7	63.6	1.0	2.5	6.2
8	82.3	2.0	1.0	2.5
9	91.4	0.0	0.0	0.6
10	100.0	0.0	0.0	0.0

Table 4

Response Patterns for the Coins and Rings Problem

Score on the TOLT	Correct response (%)	One selected from total set (%)	One selected from favorable response set (%)	
			(a)	(b)
0	0.0	8.3	41.7	25.0
1	0.0	15.4	7.1	23.1
2	14.3	21.4	0.0	21.4
3	4.7	18.6	14.0	7.0
4	15.1	24.5	5.7	26.4
5	34.1	9.8	4.9	26.8
6	37.0	8.7	7.6	12.0
7	52.4	6.0	6.0	4.8
8	63.6	1.7	0.8	7.4
9	69.8	0.0	0.0	6.0
10	100.0	0.0	0.0	0.0

Table 5

A Comparison of Response Patterns for Subjects at College, High School and Middle School : Vegetable Seeds

TOLT = 2,3

	Response Pattern					
	a-4*	b-4	b-3	D-2	Other	Total
College	13	11	4	12	14	54
High School	11	7	5	40	17	80
Middle School	16	11	9	32	14	82

χ^2 = 13.96, 8df, $p > .05$

TOLT = 4,5

	Response Pattern					
	a-4*	b-4	b-3	d-2	Other	Total
College	54	9	9	13	22	107
High School	36	8	6	20	11	81
Middle School	24	3	3	9	17	56

χ^2 = 12.15, 8df, $p > .05$

* correct response

Table 6

A Comparison of Response Patterns for Subjects at College, High School, and Middle School : Flower Seeds

TOLT = 2,3

	Response Pattern					
	d-4	b-5*	c-3	c-5	Other	Total
College	9	12	4	9	20	54
High School	21	11	8	9	21	80
Middle School	23	2	9	9	39	82

$\chi^2 = 18.4$ 8df $p < .02$

TOLT = 4,5

	Response Pattern					
	d-4	b-5*	c-3	c-5	Other	Total
College	3	59	6	7	32	107
High School	21	30	8	6	16	81
Middle School	9	18	7	4	18	56

$\chi^2 = 29.1$ 8df $p < .001$

* correct response

A gardener bought a package containing 3 squash seeds and 3 bean seeds. If just one seed is selected from the package what are the chances that it is a bean seed?

a. 1 out of 2
b. 1 out of 3
c. 1 out of 4
d. 1 out of 6
e. 4 out of 6

Reasons

1. Four selections are needed because the three squash needs could have been chosen in a row.

2. There are six seeds from which one bean seed must be chosen.

3. One bean seed needs to be selected from a total of three.

4. One half of the seeds are bean seeds.

5. In addition to a bean seed, three squash seeds could be selected from a total of six.

Figure 1. The Vegetable Seeds

An American tourist is sharing a compartment on a Swiss train with six people. Three speak only English and three speak only French. What are the chances of the tourist speaking to someone who speaks English on the first try?

a. 1 out of 2
b. 1 out of 3
c. 1 out of 4
d. 1 out of 6
e. 4 out of 6

Reasons

1. Four selections are needed because the three French speakers could be chosen in a row.

2. There are six people from which one English speaking person must be chosen.

3. One English speaking person needs to be selected from a total of three.

4. One half of the people speak English.

5. In addition to an English speaking person, three French speaking people could be selected from a total of six.

Figure 2. The Swiss Train

A gardener bought a package of 21 mixed seeds. The package contents listed:

3 short red flowers
4 short yellow flowers
5 short orange flowers
4 tall red flowers
2 tall yellow flowers
3 tall orange flowers

If just one seed is planted, what are the changes that the plant that grows will have red flowers?

a. 1 out of 2
b. 1 out of 3
c. 1 out of 7
d. 1 out of 21
e. other

Reason

1. One seed has to be chosen from among those that grow red, yellow or orange flowers.

2. 1/4 of the short and 4/9 of the talls are red.

3. It does not matter whether a tall or a short is picked. One red seed needs to be picked from a total of seven red seeds.

4. One red seed must be selected from a total of 21 seeds.

5. Seven of the twenty one seeds will produce red flowers.

Figure 3. The Flower Seeds

Three gold coins, four silver coins, and five copper coins are placed in a sack. Four gold rings, two silver rings and three copper rings are placed in the same sack.

What are the chances of pulling out a gold object on the first try?

a. 1 out of 2
b. 1 out of 3
c. 1 out of 7
d. 1 out of 21
e. none of the above

Reason

1. One gold object has to be selected from objects made from gold, silver, and copper.

2. 1/4 of the coins and 4/9 of the rings are made from gold.

3. It does not matter whether a coin or a ring is picked. One gold object needs to be selected from a total of 7 gold objects.

4. One gold object must be selected from a total of twenty-one objects.

5. 7 of the 21 objects in the sack are made from gold.

Figure 4. The Coins and the Rings